Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona’s Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The Overview acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Key Changes identifies what has been moved to and what has been moved from this particular grade level, as appropriate. This section also includes Critical Areas of Focus, which is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the Critical Areas of Focus might enable you to target specific areas of professional learning to refresh, as needed.

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the Learning Progressions. The Learning Progressions provide context for the current domain and its related standards. For any grade except Kindergarten, you will see the domain-specific standards for the current grade in the center column.

<table>
<thead>
<tr>
<th>Math Fluency Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
</tr>
<tr>
<td>1</td>
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<td>7</td>
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<td>8</td>
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</tbody>
</table>
To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the Critical Areas of Focus, these Learning Progressions can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster Description offers clarifying information, but also points to the Big Idea that can help you focus on that which is most important for this cluster within this domain. The Academic Vocabulary is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. There Deconstructed Standard for each standard specific to that cluster and each Deconstructed Standard has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the sub-standards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Instructional Targets
- Explanations and Examples

As noted, first are the Standard Statement and Standard Description, which are followed by the Essential Question(s) and the associated Mathematical Practices. The Essential Question(s) amplify the Big Idea, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the Academic Vocabulary.

The DOK Range Target for Learning and Assessment remind you of the targeted level of cognitive demand. The Instructional Targets correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the Deconstructed Standard includes Explanations and Examples. This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. Explanations and Examples may offer ideas for instructional practice and lesson plans.
# MATHEMATICS

## Standards for Mathematical Practice in 7th Grade

Each of the explanations below articulates some of the knowledge and skills expected of students to demonstrate grade-level mathematical proficiency.

<table>
<thead>
<tr>
<th>PRACTICE</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sense and persevere in problem solving.</td>
<td>Students are able to solve problems, including real-world problems, through the application of appropriate math concepts and discuss how they solved the problems. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” “Does this make sense?” and “Can I solve the problem in a different way?”</td>
</tr>
<tr>
<td>Reason abstractly and quantitatively.</td>
<td>Students represent a wide variety of real-world contexts through mathematics and can contextualize and decontextualize as needed and appropriate as they work towards a solution.</td>
</tr>
<tr>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Students construct arguments using verbal or written explanations accompanied by appropriate mathematical language and tools. Students continue to refine their mathematical communication skills through discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions such as “How did you get that?” “Why is that true?” and “Does that always work?” They explain their thinking to others and respond to others’ thinking.</td>
</tr>
<tr>
<td>Model with mathematics.</td>
<td>Students model problem situations appropriately and can create math models using expressions, equations, experiments, simulations, etc., as appropriate.</td>
</tr>
<tr>
<td>Use appropriate tools strategically.</td>
<td>Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful.</td>
</tr>
<tr>
<td>Attend to precision.</td>
<td>Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology in working through and explaining their thinking and solution.</td>
</tr>
<tr>
<td>Look for and make use of structure.</td>
<td>Students routinely seek patterns or structures to model and solve problems.</td>
</tr>
<tr>
<td>Look for and express regularity in repeated reasoning.</td>
<td>Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students formally begin to make connections and showing the relationships between concepts and in their solutions.</td>
</tr>
</tbody>
</table>
OVERVIEW

Ratios and Proportional Relationships (RP)
- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System (NS)
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations (EE)
- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry (G)
- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability (SP)
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices (MP)
- **MB 1.** Make sense of problems and persevere in solving them.
- **MB 2.** Reason abstractly and quantitatively.
- **MB 3.** Construct viable arguments and critique the reasoning of others.
- **MB 4.** Model with mathematics.
- **MB 5.** Use appropriate tools strategically.
- **MB 6.** Attend to precision.
- **MB 7.** Look for and make use of structure.
- **MB 8.** Look for and express regularity in repeated reasoning.

**KEY CHANGES**

**NEW TO 7TH GRADE**
- Constant of proportionality (7.RP.2b)
- Percent of error (7.RP.3)
- Factoring to create equivalent expressions (7.EE.1)
- Triangle side lengths (7.G.2)
- Area and circumference of circles (7.G.4)
- Angles (supplementary, complementary, vertical) (7.G.5)
- Surface area and volume of pyramids (7.G.6)
- Probability (7.SP.5 – 7.SP.8)

**MOVED FROM 7TH GRADE**
- Similar and congruent polygons (moved to 8th grade)
- Surface area and volume of cylinders (moved to 8th grade – volume only)
- Creation of box plots and histograms (moved to 6th grade – 7th grade continues to compare)
- Linear relations and functions (y-intercept moved to 8th grade)
- Views from 3-Dimensional figures (removed from CCSS)
- Statistical measures (moved to 6th grade)
### Mathematics

#### Key Changes

<table>
<thead>
<tr>
<th>New to 7th Grade</th>
<th>Moved from 7th Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>▪ Constant of proportionality (7.RP.2b)</td>
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<td></td>
</tr>
<tr>
<td>▪ Probability (7.SP.5 – 7.SP.8)</td>
<td></td>
</tr>
</tbody>
</table>
1. Developing understanding of and applying proportional relationships.
   • Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Developing understanding of operations with rational numbers and working with expressions and linear equations.
   • Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.
   • Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

4. Drawing inferences about populations based on samples.
   • Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
DOMAIN:

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)

SEVENTH GRADE

MATHEMATICS
## Ratios and Proportional Relationships (RP)

### Clusters

1. Analyze proportional relationships and use them to solve real-world and mathematical problems.

### Ratios and Proportional Relationships (RP)

<table>
<thead>
<tr>
<th></th>
<th>Sixth</th>
<th>Seventh</th>
<th>Eighth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio Boxes and Unit Ratio/Rate</td>
<td>6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td>
<td>7.RP.2.b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
<td>7.RP.2.c Represent proportional relationships by equations.</td>
</tr>
<tr>
<td></td>
<td>6.RP.2 Understand the concept of a unit rate (a/b) associated with a ratio (ab) with (b \neq 0), and use rate language in the context of a ratio relationship.</td>
<td>7.RP.2.c Represent proportional relationships by equations.</td>
<td>7.RP.2.d Explain what a point ((x, y)) on the graph of a proportional relationship means in terms of the situation, with special attention to the points ((0, 0)) and ((1, r)) where (r) is the unit rate.</td>
</tr>
<tr>
<td></td>
<td>7.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed.</td>
<td>7.RP.2.d Explain what a point ((x, y)) on the graph of a proportional relationship means in terms of the situation, with special attention to the points ((0, 0)) and ((1, r)) where (r) is the unit rate.</td>
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</tr>
<tr>
<td></td>
<td>7.RP.3.a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td>7.RP.2.a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</td>
<td>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.</td>
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</table>

### Graphing Proportional Relationships

- **Sixth**: Graphing Proportional Relationships
- **Seventh**: Graphing Proportional Relationships
- **Eighth**: Graphing Proportional Relationships

<table>
<thead>
<tr>
<th>Percent</th>
<th>Sixth</th>
<th>Seventh</th>
<th>Eighth</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.RP.3.c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</td>
<td>7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.</td>
<td></td>
<td>8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</td>
</tr>
</tbody>
</table>

### Percents

- **Sixth**: Percents
- **Seventh**: Percents
- **Eighth**: Percents

<table>
<thead>
<tr>
<th>Percent</th>
<th>Sixth</th>
<th>Seventh</th>
<th>Eighth</th>
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<tbody>
<tr>
<td>6.EE.2.c Evaluate expressions at specific values of their variables. Include expressions embedded in formulas or equations from real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</td>
<td>6.EE.3 Apply the properties of operations to generate equivalent expressions.</td>
<td></td>
<td>6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).</td>
</tr>
<tr>
<td>6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td>
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</tbody>
</table>
# CLUSTER
1. Analyze proportional relationships and use them to solve real-world and mathematical problems.

## BIG IDEA
- Numbers are compared by their relative value.

## ACADEMIC VOCABULARY
- unit rates, ratios, proportional relationships, proportions, constant of proportionality, complex fractions

## STANDARD AND DECONSTRUCTION

### 7.RP.1
**Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour.**

**DESCRIPTION**
Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. The comparison can be with like or different units. Fractions may be proper or improper.

**Example 1:**
If $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then how much paint is needed for the entire wall?

**Solution:**
$\frac{1}{2}$ gal / $\frac{1}{6}$ wall.
$\frac{3}{2}$ gallons per 1 wall

## ESSENTIAL QUESTION(S)
How can ratio and rate reasoning be used to efficiently solve real-world problems?

## MATHEMATICAL PRACTICE(S)
- 7.MP.2. Reason abstractly and quantitatively.

## DOK RANGE TARGET FOR INSTRUCTION & ASSESSMENT
- **1**
- **2**
- **3**
- **4**

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Compute unit rates associated with ratios of fractions in like or different units.</td>
<td>Evaluate expressions using the order of operations (including using parentheses, brackets, or braces).</td>
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</tbody>
</table>
The really nice thing about this standard is that students won’t be as tempted to ask, “Why do I need to know this?” The standard is all about real world situations: taxes, tips, sports, cooking, shopping, building, scientific principles, other mathematical principles... and much, much more. The following examples will demonstrate how those real world problems should look as you provide students opportunities to explore concepts related to the content standard using the mathematical process standards.

A variety of visual tools will help your students understand the relationship between ratios and proportional rates in multistep problems. Examples of visuals include tables, double number lines, graphs, and tape diagrams.

**Table:**
(Consider also using a function table.)

Consider the following problems (MP4.5,7,8):

“If a recipe for a dozen cookies calls for \(\frac{1}{4}\) cup of brown sugar and \(\frac{1}{2}\) cup of white sugar, how much brown sugar will you need for 5 dozen cookies?”

*The table allows students to see that for 5 dozen cookies, you will need \(\frac{1}{4}\) cups of brown sugar in order to bake 5 dozen cookies. This answer demonstrates the idea of equivalency within rates.*

“What is the equivalent unit rate of \(\frac{1}{5}\) of cup of white sugar?”

*The table shows that \(\frac{1}{2}\) cup of brown sugar is needed per cup of white sugar*

**Tables: Ratio of Amount of Brown Sugar to Amount of White Sugar Per Dozen as a Proportional Rate**

<table>
<thead>
<tr>
<th>Dozens</th>
<th>1 dozen</th>
<th>2 dozen</th>
<th>3 dozen</th>
<th>4 dozen</th>
<th>5 dozen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups Brown Sugar</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{3}{4})</td>
<td>(\frac{4}{4})</td>
<td>(\frac{5}{4})</td>
</tr>
<tr>
<td>Cups White Sugar</td>
<td>(\frac{1}{2})</td>
<td>(\frac{2}{2})</td>
<td>(\frac{3}{2})</td>
<td>(\frac{4}{2})</td>
<td>(\frac{5}{2})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dozens</th>
<th>Cups of Brown Sugar</th>
<th>Cups of White Sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{4})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>(\frac{2}{2})</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{3}{4})</td>
<td>(\frac{3}{2})</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{4}{4})</td>
<td>(\frac{4}{2})</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{5}{4})</td>
<td>(\frac{5}{3})</td>
</tr>
</tbody>
</table>
Consider the following problems:

“If a farmer harvests \( \frac{1}{8} \) of his crop every 2 \( \frac{1}{2} \) days, how long will it take him to harvest his entire crop?”

*The double number line allows students to see that in order to harvest the entire crop at a constant rate, it will take 12 days.

“What is the equivalent unit rate of \( \frac{1/8}{5/2} \) per day?”

*Since the number line does not show a direct relationship between the amount of crop harvested in a single day, students can use their reasoning skills to persevere at solving this problem (MP1,2), or they can make use of a standard algorithm, whereby the student should simply divide \( \frac{10}{5} \) to arrive at the unit rate of \( \frac{1}{6} \) of the crop harvested per day. This will look like \( \frac{1}{6} \times \frac{2}{8} = \frac{2}{40} = \frac{1}{20} \) of the crop per day. *

**Double Number Line:**

<table>
<thead>
<tr>
<th>( \frac{1}{8} )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Crop Harvested</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 ( \frac{1}{2} )</td>
<td>3</td>
<td>4 ( \frac{1}{2} )</td>
<td>6</td>
<td>7 ( \frac{1}{2} )</td>
<td>9</td>
<td>10 ( \frac{1}{2} )</td>
<td>12</td>
</tr>
</tbody>
</table>

**Graph:**

“If a champion swimmer consistently swims \( \frac{3}{2} \) of a length of an Olympic-sized pool in \( \frac{1}{10} \) minute, how long will it take him to swim 4 lengths of the pool?”

*The line graph allows students to see that it would take a champion swimmer about \( \frac{8}{10} \) of a minute to swim 4 lengths of the pool.*

“What is the equivalent unit rate of \( \frac{3/2}{1/10} \) per minute?”

*Again, since the graph does not show the number of lengths one would swim in one minute, students could reason quantitatively (MP2) in order to persevere in solving this question (MP1). Otherwise, an algorithm could be used to compute the equivalent unit rate. This would look like \( \frac{1}{2} \times \frac{10}{1} = \frac{10}{2} = 5 \) lengths per minute.*

**Line Graph:**

Ratio of Length of Pool Swam to Fraction/Decimal of a Minute as a Proportional Rate

- 0.1
- 0.2
- 0.3
- 0.4
- 0.5
- 0.6
- 0.7
- 0.8
- 0.9
- 1

Minutes

1/2 length | 1 length | 1 1/2 length | 2 lengths | 2 1/2 lengths | 3 lengths | 3 1/2 lengths | 4 lengths
Consider the following problems:

“If an avid reader reads $\frac{1}{2}$ of a 400 page book every $1 \frac{1}{2}$ days, how many days will it take for that person to read three approximately 400-page books?"

*The tape diagram clearly shows that it would take 6 days to read those three books."

“What is the equivalent unit rate of $\frac{200}{3/2}$ pages per day?"

*Since the tape diagram does not show a direct relationship between the number of pages read in a single day, again, students can use their reasoning skills to persevere at solving this problem (MP1,2), or they can make use of a standard algorithm, whereby the student could simply divide $\frac{200}{3/2}$ to arrive at the unit rate of pages read per day. This will look like $\frac{200}{1} \times \frac{2}{3} = \frac{400}{3}$, or $\sim 133$ pages per day."

**Tape Diagram:**

<table>
<thead>
<tr>
<th>200 pages</th>
<th>400 pages</th>
<th>800 pages</th>
<th>1000 pages</th>
<th>1200 pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$ days</td>
<td>3 days</td>
<td>$4 \frac{1}{2}$ days</td>
<td>6 days</td>
<td>$4 \frac{1}{2}$ days</td>
</tr>
</tbody>
</table>

Stephanie Chiarelli, CCLM^2 Project, Summer, 2012

This material was developed as part of the Leadership for the Common Core in Mathematics (CCLM^2) project at the University of Wisconsin, Milwaukee.
STANDARD AND DECONSTRUCTION

7.RP.2 Recognize and represent proportional relationships between quantities.

DESCRIPTION

Students’ understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students determine if two quantities are in a proportional relationship from a table. Fractions and decimals could be used with this standard.

Note: This standard focuses on the representations of proportions. Solving proportions is addressed in 7.SP.3.

Example 1:
The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship?

<table>
<thead>
<tr>
<th>Number of Books</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
</tr>
</tbody>
</table>

Solution:
Students can examine the numbers to determine that the price is the number of books multiplied by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table; therefore, the table does not represent a proportional relationship.

Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books, 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when x = 1 will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.
Example 2:
The graph below represents the price of the bananas at one store. What is the constant of proportionality?

Solution:
From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph. Note: Any point on the line will yield this constant of proportionality.

Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality.

Example 3:
The price of bananas at another store can be determined by the equation: \( P = 0.35n \), where \( P \) is the price and \( n \) is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Solution:
The constant of proportionality is the coefficient of \( x \) (or the independent variable). The constant of proportionality is 0.35.

ESSENTIAL QUESTION(S)
What mathematical language and representation can be used when examining proportional relationships?

MATHEMATICAL PRACTICE(S)
7.MP.1. Make sense of problems and persevere in solving them.
7.MP.2. Reason abstractly and quantitatively.
7.MP.3. Construct viable arguments and critique the reasoning of others.
7.MP.5. Use appropriate tools strategically.
7.MP.6. Attend to precision.
7.MP.7. Look for and make use of structure.
7.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>
### 7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Know that a proportion is a statement of equality between two ratios.</td>
<td>Analyze two ratios to determine if they are proportional to one another with a variety of strategies (e.g., using tables, graphs, pictures, etc.).</td>
<td></td>
</tr>
</tbody>
</table>

### 7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
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<th>Do</th>
</tr>
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<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Define a constant of proportionality as a unit rate.</td>
<td>Analyze tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships to identify the constant of proportionality.</td>
<td></td>
</tr>
</tbody>
</table>

### 7.RP.2c Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t = pn$.

<table>
<thead>
<tr>
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<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Represent proportional relationships by writing equations.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 7.RP.2d Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

<table>
<thead>
<tr>
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<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Recognize what $(0, 0)$ represents on the graph of a proportional relationship. Recognize what $(1, r)$ on a graph represents, where $r$ is the unit rate.</td>
<td>Explain what the points on a graph of a proportional relationship mean in terms of a specific situation.</td>
<td></td>
</tr>
</tbody>
</table>
Students may use a content website and/or interactive whiteboard to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.

**Examples:**
- A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

<table>
<thead>
<tr>
<th>Serving Size</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Nuts (x)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Cups of Fruit (y)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1).

The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.
- The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

<table>
<thead>
<tr>
<th>Number of Packs of Gum (g)</th>
<th>Cost in Dollars (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Equation: \(2g = d\), where \(d\) is the cost in dollars and \(g\) is the packs of gum. A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using \(x\) and \(y\). Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps students revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost.

The really nice thing about this standard is that students won’t be as tempted to ask, “Why do I need to know this?” The standard is all about real world situations: taxes, tips, sports, cooking, shopping, building, scientific principles, other mathematical principles... and much, much more. The following examples will demonstrate how those real world problems should look as you provide students opportunities to explore concepts related to the content standard using the mathematical process standards.

A variety of visual tools will help your students understand the relationship between ratios and proportional rates in multistep problems. Examples of visuals include tables, double number lines, graphs, and tape diagrams.
Table:

<table>
<thead>
<tr>
<th>Number of Packs of Gum (g)</th>
<th>Cost in Dollars (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
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Equation: \(2g = d\), where \(d\) is the cost in dollars and \(g\) is the packs of gum.

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using \(x\) and \(y\). Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars.” They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps students revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost.
7.RP.3

**DESCRIPTION**

In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication.

Students understand the mathematical foundation for cross-multiplication. An explanation of this foundation can be found in Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

**Example 1:**

Sally has a recipe that needs \( \frac{3}{4} \) teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed?

Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

\[
\frac{\frac{3}{4}}{2} = \frac{x}{3}
\]

**Solution:**

One possible solution is to recognize that \( 2 \times 1 \frac{1}{2} = 3 \) so \( \frac{3}{4} \times 1 \frac{1}{2} = x \). The amount of butter needed would be \( 1 \frac{1}{8} \) teaspoons.

A second way to solve this proportion is to use cross-multiplication \( \frac{3}{4} \times 3 = 2x \). Solving for \( x \) would give \( 1 \frac{1}{8} \) teaspoons of butter.

Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error. (Note the similarity between percent error and percent of increase or decrease).

\[
\text{% error} = \frac{|\text{estimated value} - \text{actual value}|}{\text{actual value}} \times 100\%
\]

**Example 2:**

Jamal needs to purchase a countertop for his kitchen. Jamal measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is Jamal's percent error?

**Solution:**

\[
\text{% error} = \frac{|5 \text{ ft} - 4.5 \text{ ft}|}{4.5} \times 100\%
\]

\[
\text{% error} = \frac{0.5 \text{ ft}}{4.5} \times 100\%
\]
The use of proportional relationships is also extended to solve percent problems involving sales tax, markups and markdowns simple interest ($I = prt$, where $I =$ interest, $p =$ principal, $r =$ rate, and $t =$ time (in years)), gratuities and commissions, fees, percent increase and decrease, and percent error.

For example, Games Unlimited buys video games for $10. The store increases their purchase price by 300%. What is the sales price of the video game? Using proportional reasoning, if $10$ is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30$ would be $10$ times 3. Thirty dollars represents the amount of increase from $10$ so the new price of the video game would be $40$.

Example 3:
Stephanie paid $9.18 for a pair of earrings. This amount includes a tax of 8%. What was the cost of the item before tax?

Solution:
One possible solution path follows:
$9.18$ represents 100% of the cost of the earrings + 8% of the cost of the earrings. This representation can be expressed as $1.08c = 9.18$, where $c$ represents the cost of the earrings. Solving for $c$ gives $8.50$ for the cost of the earrings.

Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

<table>
<thead>
<tr>
<th>ESSENTIAL QUESTION(S)</th>
<th>How can proportional reasoning be used to efficiently solve real world problems involving rational numbers?</th>
</tr>
</thead>
</table>
| MATHEMATICAL PRACTICE(S) | 7.MP.1. Make sense of problems and persevere in solving them.  
7.MP.2. Reason abstractly and quantitatively.  
7.MP.3. Construct viable arguments and critique the reasoning of others.  
7.MP.5. Use appropriate tools strategically.  
7.MP.6. Attend to precision.  
7.MP.7. Look for and make use of structure.  
7.MP.8. Look for and express regularity in repeated reasoning. |

<table>
<thead>
<tr>
<th>DOK Range Target for Instruction &amp; Assessment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</table>

<table>
<thead>
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<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
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</tr>
<tr>
<td>Students should be able to:</td>
<td>Recognize situations in which proportional relationships apply.</td>
<td>Apply proportional reasoning to solve multi-step ratio and percent problems.</td>
<td></td>
</tr>
</tbody>
</table>
Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer aew reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

Examples:

• Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say: “The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40.”

$4.17 + 4.17 + (0.24 \cdot 4.17) = 2.24 \times 4.17$

<table>
<thead>
<tr>
<th>100%</th>
<th>100%</th>
<th>24%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.17$</td>
<td>$4.17$</td>
<td>?</td>
</tr>
</tbody>
</table>

• A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

<table>
<thead>
<tr>
<th>37.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>33% of 37.50</td>
</tr>
</tbody>
</table>

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 \times \text{Original Price}.

Continued on next page

• A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

<table>
<thead>
<tr>
<th>Discount</th>
<th>Sale Price - $12</th>
<th>Original Price (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% of original price</td>
<td>60% of original price</td>
<td>0.60p = 12</td>
</tr>
</tbody>
</table>

• At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.

• A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500, as well as, a 10% commission for all sales. How much merchandise will he have to sell to meet his goal?

After eating at a restaurant, your bill before tax is $52.60. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

The amount paid = 0.20 \times $52.50 + 0.08 \times $52.50 = 0.28 \times $52.50
THE NUMBER SYSTEM
(NS)

SEVENTH GRADE
MATHEMATICS
## SEVENTH GRADE

**LEXILE GRADE LEVEL BANDS: 970L TO 1120L**

### DOMAIN

**The Number System (NS)**

### CLUSTERS

1. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### THE NUMBER SYSTEM (NS)

#### SIXTH

<table>
<thead>
<tr>
<th>Integers on the Number Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</td>
</tr>
</tbody>
</table>

#### SEVENTH

<table>
<thead>
<tr>
<th>Integers on the Number Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.NS.1.a Describe situations in which opposite quantities combine to make 0.</td>
</tr>
</tbody>
</table>

#### EIGHTH

<table>
<thead>
<tr>
<th>Integers on the Number Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
</tr>
</tbody>
</table>

### INTEGERS, NUMBER LINES, AND COORDINATE PLANES

<table>
<thead>
<tr>
<th>6.NS.5</th>
<th>7.NS.1.a</th>
<th>8.G.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.</td>
<td>Describe situations in which opposite quantities combine to make 0.</td>
<td>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.NS.6.a</th>
<th>6.NS.7.a</th>
<th>6.NS.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite.</td>
<td>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.</td>
<td>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</td>
<td>Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.NS.7.d</th>
<th>6.NS.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguish comparisons of absolute value from statements about order.</td>
<td>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td>
</tr>
</tbody>
</table>
## Mathematics

### The Number System (NS)

<table>
<thead>
<tr>
<th>6th Grade</th>
<th>7th Grade</th>
<th>8th Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rational and Irrational Numbers</strong></td>
<td><strong>Rational and Irrational Numbers</strong></td>
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</tr>
<tr>
<td><strong>Adding and Subtracting Rational Numbers</strong></td>
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</tr>
<tr>
<td>6.NS.7.b Write, interpret, and explain statements of order for rational numbers in real-world contexts.</td>
<td>7.NS.1.b Understand $p + q$ as the number located a distance $</td>
<td>q</td>
</tr>
<tr>
<td>6.NS.6.c Find and position integers and other rational numbers on a horizontal or vertical number line diagram.</td>
<td>7.NS.1.c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</td>
<td></td>
</tr>
<tr>
<td><strong>Multiplying and Dividing Rational Numbers</strong></td>
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<td><strong>Multiplying and Dividing Rational Numbers</strong></td>
</tr>
<tr>
<td>7.NS.2.a Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</td>
<td>7.NS.2.b Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</td>
<td></td>
</tr>
<tr>
<td>7.NS.2.c Apply properties of operations as strategies to multiply and divide rational numbers.</td>
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<td></td>
</tr>
<tr>
<td>7.NS.3 Solve real-world problems using the four operations with rational numbers (including complex fractions).</td>
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<td></td>
</tr>
<tr>
<td>7.NS.2.d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</td>
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</tr>
<tr>
<td>7.EE.3 Solve multi-step real-world problems coordinating among integers, fractions, decimals, and percents.</td>
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<td></td>
</tr>
</tbody>
</table>
### SEVENTH GRADE

**LEXILE GRADE LEVEL BANDS: 970L TO 1120L**

#### CLUSTER

1. **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**

#### BIG IDEA

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- The value of any real number can be determined by applying the operations of addition, subtraction, multiplication or division of two or more specific numbers.

#### ACADEMIC VOCABULARY

- proportion
- rational number
- ratio
- reciprocal
- least common multiple
- inverse operation
## 7.NS.1

<table>
<thead>
<tr>
<th>STANDARD AND DECONSTRUCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</strong></td>
</tr>
</tbody>
</table>

### DESCRIPTION

Students add and subtract rational numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the CCSS is to build on students’ understandings of number lines developed in 6th grade.

**Example 1:**

Use a number line to add $$-5 + 7$$.

**Solution:**

Students find $$-5$$ on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts.

In 6th grade, students found the distance of horizontal and vertical segments on the coordinate plane. In 7th grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line.

In the example, $$7 - 5$$, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2.

**Example 2:**

Use a number line to subtract: $$-6 - (-4)$$.

**Solution:**

This problem is asking for the distance between $$-6$$ and $$-4$$. The distance between $$-6$$ and $$-4$$ is 2 and the direction from $$-4$$ to $$-6$$ is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of $$-4$$: $$-6 + 4 = -2$$. 

---

**NUMBER SYSTEM**
## Essential Question(S)

How can previous mathematical understanding of number operations apply to adding and subtracting rational numbers?

## Mathematical Practice(S)

- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.7. Look for and make use of structure.

## DOK Range Target for Instruction & Assessment

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## Instructional Targets

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
<td></td>
</tr>
</tbody>
</table>

### Students should be able to:

- Describe situations in which opposite quantities combine to make 0.
- Represent and explain how a number and its opposite have a sum of 0 and are additive inverses.
- Demonstrate and explain how adding two numbers, \( p + q \), if \( q \) is positive, the sum of \( p \) and \( q \) will be \( |q| \) spaces to the right of \( p \) on the number line.
- Demonstrate and explain how adding two numbers, \( p + q \), if \( q \) is negative, the sum of \( p \) and \( q \) will be \( |q| \) spaces to the left of \( p \) on the number line.
- Identify subtraction of rational numbers as adding the additive inverse property to subtract rational numbers, \( p - q = p + (-q) \).
- Identify properties of addition and subtraction when adding and subtracting.
- Interpret sums of rational numbers by describing real-world contexts.
- Explain and justify why the sum of \( p + q \) is located a distance of \( |q| \) in the positive or negative direction from \( p \) on a number line.
- Apply and extend previous understanding to represent addition and subtraction problems of rational numbers with a horizontal or vertical number line.
- Apply and extend previous understanding to represent addition and subtraction problems of rational numbers with a horizontal or vertical number line.
- Represent the distance between two rational numbers on a number line is the absolute value of their difference and apply this principle in real-world contexts.
- Represent the distance between two rational numbers on a number line is the absolute value of their difference and apply this principle in real-world contexts.
- Apply the principle of subtracting rational numbers in real-world contexts.
- Apply properties of operations as strategies to add and subtract rational numbers.
### 7.NS.1a Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

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<td><strong>Students should be able to:</strong></td>
<td>Describe situations in which opposite quantities combine to make 0.</td>
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</table>

### 7.NS.1b Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

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<tr>
<td><strong>Students should be able to:</strong></td>
<td>Represent and explain how a number and its opposite have a sum of 0 and are additive inverses. Demonstrate and explain how adding two numbers, p + q, if q is positive, the sum of p and q will be</td>
<td>q</td>
<td>spaces to the right of p on the number line. Demonstrate and explain how adding two numbers, p + q, if q is negative, the sum of p and q will be</td>
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</table>

### 7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
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<tr>
<td><strong>Students should be able to:</strong></td>
<td>Identify subtraction of rational numbers as adding the additive inverse property to subtract rational numbers, p - q = p + (-q).</td>
<td>Apply and extend previous understanding to represent addition and subtraction problems of rational numbers with a horizontal or vertical number line. Represent the distance between two rational numbers on a number line is the absolute value of their difference and apply this principle in real-world contexts. Apply the principle of subtracting rational numbers in real-world contexts.</td>
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</tbody>
</table>
**7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.**

**Instructional Targets**
- **Know:** Concepts/Skills
  - Tasks assessing concepts, skills, and procedures.
- **Think**
  - Tasks assessing expressing mathematical reasoning.
- **Do**
  - Tasks assessing modeling/applications.

**Assessment Types**
- Students should be able to:
  - Identify properties of addition and subtraction when adding and subtracting.
  - Apply properties of operations as strategies to add and subtract rational numbers.

**Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.**

**Examples:**
- Use a number line to illustrate:
  - $p - q$
  - $p + (-q)$
  - Is this equation true $p - q = p + (-q)$?

- $-3$ and $3$ are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and its opposite is zero.

- You have $4$ and you need to pay a friend $3$. What will you have after paying your friend?
  - $4 + (-3) = 1$ or $(-3) + 4 = 1$
STANDARD AND DECONSTRUCTION

**7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

**DESCRIPTION**

Students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign.

**Example 1:**
Which of the following fractions is equivalent to $\frac{-4}{5}$? Explain your reasoning.

a. $\frac{4}{-5}$  
b. $\frac{-16}{20}$  
c. $\frac{-4}{-5}$

Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for the work with rational and irrational numbers in 8th grade.

**Example 2:**
Using long division, express the following fractions as decimals. Which of the following fractions will result in terminating decimals; which will result in repeating decimals?
Identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5).

**ESSENTIAL QUESTION(S)**

How can previous mathematical understanding of number operations apply to multiplying and dividing rational numbers?

**MATHEMATICAL PRACTICE(S)**

7.MP.2. Reason abstractly and quantitatively.
7.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

| 1 | 2 | 3 | 4 |

**SUBSTANDARD DECONSTRUCTED**

7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

**Instructional Targets**

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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Recognize that the process for multiplying fractions can be used to multiply rational numbers including integers. Know and describe the rules when multiplying signed numbers.</td>
<td>Apply the properties of operations, particularly distributive property, to multiply rational numbers. Interpret the products of rational numbers by describing real-world contexts.</td>
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</tbody>
</table>
### 7.NS.2b Understand that integers can be divided provided that the divisor is not zero and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then \(-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}\). Interpret quotients of rational numbers by describing real-world contexts.

**Instructional Targets**

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<tbody>
<tr>
<td>Students should be able to:</td>
<td>Explain why integers can be divided except when the divisor is 0.</td>
<td>Interpret the quotient of rational numbers by describing real-world contexts.</td>
</tr>
<tr>
<td></td>
<td>Describe why the quotient is always a rational number.</td>
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<tr>
<td></td>
<td>Know and describe the rules when dividing signed numbers, integers.</td>
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<tr>
<td></td>
<td>Recognize that (-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}).</td>
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</table>

### 7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

**Instructional Targets**

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<td>Tasks assessing expressing mathematical reasoning.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Identify how properties of operations can be used to multiply and divide rational numbers.</td>
<td>Apply properties of operations as strategies to multiply and divide rational numbers.</td>
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</table>

### 7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

**Instructional Targets**

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<tr>
<td>Assessment Types</td>
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<td>Tasks assessing expressing mathematical reasoning.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Convert a rational number to a decimal using long division.</td>
<td>Explain that the decimal form of a rational number terminates (stops) in zeroes or repeats.</td>
</tr>
</tbody>
</table>
Multiplication and division of integers is an extension of multiplication and division of whole numbers.

**Examples:**

- Examine the family of equations. What patterns do you see? Create a model and context for each of the products.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Number Line Model</th>
<th>Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 x 3 = 6</td>
<td><img src="image1" alt="Number Line Model" /></td>
<td>Selling two posters at $3.00 per poster</td>
</tr>
<tr>
<td>2 x -3 = -6</td>
<td><img src="image2" alt="Number Line Model" /></td>
<td>Spending 3 dollars each on 2 posters</td>
</tr>
<tr>
<td>-2 x 3 = -6</td>
<td><img src="image3" alt="Number Line Model" /></td>
<td>Owing 2 dollars to each of your three friends</td>
</tr>
<tr>
<td>-2 x -3 = 6</td>
<td><img src="image4" alt="Number Line Model" /></td>
<td>Forgiving 3 debts of $2.00 each</td>
</tr>
</tbody>
</table>
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.NS.3</th>
<th>Solve real-world and mathematical problems involving the four operations with rational numbers.¹</th>
</tr>
</thead>
</table>

#### DESCRIPTION
Students use order of operations from 6th grade to write and solve problems with all rational numbers.

**Example 1:**
Calculate: \([-10(-0.9)] - [(-10) \cdot 0.11]\)

**Solution:** 10.1

**Example 2:**
Jim’s cell phone bill is automatically deducting $32 from his bank account every month. How much will the deductions total for the year?

**Example 3:**
A newspaper reports these changes in the price of a stock over four days: -1/8, -5/8, 3/8, -9/8. What is the average daily change?

**Solution:**
The sum is -12/8; dividing by 4 will give a daily average of -3/8.

### ESSENTIAL QUESTION(S)
What strategies can be used to accurately solve real world and mathematical problems involving rational numbers?

### MATHEMATICAL PRACTICE(S)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment

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<tr>
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</tr>
</tbody>
</table>

**Students should be able to:**
- Add, subtract, multiply, and divide rational numbers.
- Solve real-world mathematical problems by adding, subtracting, multiplying, and dividing rational numbers, including complex fractions.

¹ Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
Examples:

- Your cell phone bill is automatically deducting $32 from your bank account every month. How much will the deductions total for the year?
  

- It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?
  
  \[
  \frac{-100 \text{ feet}}{20 \text{ seconds}} = \frac{-5 \text{ feet}}{1 \text{ second}} = -5 \text{ ft/sec}
  \]
DOMAIN:

EXPRESSIONS AND EQUATIONS (EE)

SEVENTH GRADE MATHEMATICS
## Expressions and Equations (EE)

1. Use properties of operations to generate equivalent expressions.
2. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

### Expressions and Equations (EE)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Expressions and Equations (EE)</th>
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<tbody>
<tr>
<td>Clusters</td>
<td>Working with Expressions</td>
</tr>
<tr>
<td>Sixth</td>
<td>6.EE.2.a Write expressions that record operations with numbers and with letters standing for numbers.</td>
</tr>
<tr>
<td>Seventh</td>
<td>6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td>
</tr>
<tr>
<td>Eighth</td>
<td>7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</td>
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</table>
# MATHEMATICS

## EXPRESSIONS AND EQUATIONS (EE)

<table>
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<th>SIXTH</th>
<th>SEVENTH</th>
<th>EIGHTH</th>
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<tbody>
<tr>
<td><strong>LINEAR EQUATIONS, INEQUALITIES, AND FUNCTIONS</strong></td>
<td><strong>Solving Linear Equations and Inequalities</strong></td>
<td><strong>Solving Linear Equations and Inequalities</strong></td>
</tr>
<tr>
<td><strong>6.EE.5</strong> Understand solving an equation or inequality as a process of answering a question which values from a specified set, if any, make the equation or inequality true. Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</td>
<td><strong>7.EE.4.a</strong> Solve word problems leading to equations of the form ( px + q = r ) and ( p(x + q) = r ), where ( p, q, ) and ( r ) are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</td>
<td><strong>8.EE.7.b</strong> Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</td>
</tr>
<tr>
<td><strong>6.EE.8</strong> Write an inequality of the form ( x &gt; c ) or ( x &lt; c ) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form ( x &gt; c ) or ( x &lt; c ) have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</td>
<td><strong>7.EE.4.b</strong> Solve word problems leading to inequalities of the form ( px + q &gt; r ) or ( px + q &lt; r ), where ( p, q, ) and ( r ) are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</td>
<td><strong>8.EE.7.a</strong> Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form ( x = a, a = a, ) or ( a = b ) results (where ( a ) and ( b ) are different numbers).</td>
</tr>
<tr>
<td><strong>6.EE.7</strong> Solve real-world and mathematical problems by writing and solving equations of the form ( x + p = q ) and ( px = q ) for cases in which ( p, q, ) and ( x ) are all nonnegative rational numbers.</td>
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</tbody>
</table>
1. Use properties of operations to generate equivalent expressions.

**BIG IDEA**
- There are infinite ways to express a number or expression.

**ACADEMIC VOCABULARY**
coefficients, like terms, distributive property, factor

### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.EE.1</th>
<th>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DESCRIPTION</strong></td>
<td>This is a continuation of work from 6th grade using properties of operations and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive / negative fractions and decimals) to write equivalent expressions.</td>
</tr>
<tr>
<td><strong>Example 1:</strong></td>
<td>What is the length and width of the rectangle below?</td>
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<table>
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<tbody>
<tr>
<td>2a</td>
<td>4b</td>
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</table>

| **Solution:** | The Greatest Common Factor (GCF) is 2, which will be the width because the width is in common to both rectangles. To get the area 2a multiply by a, which is the length of the first rectangles. To get the area of 4b, multiply by 2b, which will be the length of the second rectangle. The final answer will be 2(a + 2b). |
| **Example 2:** | Write an equivalent expression for 3(x + 5) – 2. |
| **Solution:** | 3x + 15 – 2 Distribute the 3 3x + 13 Combine like terms |
## ESSENTIAL QUESTION(S)

What strategies can be applied to add, subtract, factor and expand linear equations?

## MATHEMATICAL PRACTICE(S)

- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.7. Look for and make use of structure.

## DOK Range Target for Instruction & Assessment

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## Instructional Targets

### Assessment Types

- Tasks assessing concepts, skills, and procedures.
- Tasks assessing expressing mathematical reasoning.
- Tasks assessing modeling/applications.

### Students should be able to:

- Combine like terms with rational coefficients.
- Factor and expand linear expressions with rational coefficients using the distributive property.
- Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## EXPLANATIONS AND EXAMPLES

### Examples:

- Write an equivalent expression for $3(x + 5) - 2$.
- Suzanne thinks the two expressions $2(3a - 2) + 4a$ and $10a - 2$ are equivalent? Is she correct? Explain why or why not?
- Write equivalent expressions for: $3a + 12$.

Possible solutions might include factoring as in $3(a + 4)$, or other expressions such as $a + 2a + 7 + 5$.

A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w + w + 2w + 2w$. Write the expression in two other ways.

**Solution:** $6w$ or $2(w) + 2(2w)$.

- An equilateral triangle has a perimeter of $6x + 15$. What is the length of each of the sides of the triangle?

**Solution:** $3(2x + 5)$, therefore each side is $2x + 5$ units long.
### STANDARD AND DECONSTRUCTION

| 7.EE.2 | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”* |

#### DESCRIPTION
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, \( a + 0.05a = 1.05a \) means that “increase by 5%” is the same as “multiply by 1.05.”

**Example:**
All varieties of a certain brand of cookies are $3.50. A person buys peanut butter cookies and chocolate chip cookies. Write an expression that represents the total cost, \( T \), of the cookies if \( p \) represents the number of peanut butter cookies and \( c \) represents the number of chocolate chip cookies.

**Solution:**
Students could find the cost of each variety of cookies and then add to find the total.
\[
T = 3.50p + 3.50c
\]
Or students could recognize that multiplying 3.50 by the total number of boxes (regardless of variety) will give the same total.
\[
T = 3.50(p + c)
\]

#### ESSENTIAL QUESTION(S)
Why is examining algebraic expressions in different forms important?

#### MATHEMATICAL PRACTICE(S)
7.MP.2. Reason abstractly and quantitatively.
7.MP.6. Attend to precision.
7.MP.7. Look for and make use of structure.
7.MP.8. Look for and express regularity in repeated reasoning.

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#### Instructional Targets
**Know: Concepts/Skills**
Tasks assessing concepts, skills, and procedures.

**Think**
Tasks assessing expressing mathematical reasoning.

**Do**
Tasks assessing modeling/applications.

**Assessment Types**

**Students should be able to:**
Generate equivalent fractions to find like denominators.

Rewrite an expression in an equivalent form in order to provide insight about how quantities are related in a problem context.
Examples:

• Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if \( J \) = the number of hours that Jamie worked this week and \( T \) = the number of hours Ted worked this week? Can you write the expression in another way?

Students may create several different expressions depending upon how they group the quantities in the problem.

One student might say: “To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the $27 overtime to find the total wages for the week.” The student would write the expression \( 9J + 9T + 27 \).

Another student might say: “To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week.” The student would write the expression \( 9(J + T) + 27 \).

A third student might say: “To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie’s wages, I would multiply the number of hours she worked by 9. To figure out Ted’s wages, I would multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages.” The student would write the expression \( (9J) + (9T + 27) \).

• Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrates that the expressions are equivalent. Which expression do you think is most useful? Explain your thinking.
2. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- Real world problems are solved using the four operations, formulas, plots and units of measurement.

numeric expressions, algebraic expressions, maximum, minimum

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Students solve contextual problems and mathematical problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers.

Example:
Three students conduct the same survey about the number of hours people sleep at night. The results of the number of people who sleep 8 hours a night are shown below. In which person's survey did the most people sleep 8 hours?

- Susan reported that 18 of the 48 people she surveyed get 8 hours sleep a night.
- Kenneth reported that 36% of the people he surveyed get 8 hours sleep a night.
- Jamal reported that 0.365 of the people he surveyed get 8 hours sleep a night.

Solution:
In Susan's survey, the number is 37.5%, which is the greatest percentage.

What efficient strategies can be used for solving real-life and mathematical problems involving positive and negative rational numbers in any form?

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
### Mathematics

#### DOK Range Target for Instruction & Assessment

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<td>Instructional Targets</td>
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<tr>
<td>Assessment Types</td>
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<tr>
<td>Students should be able to:</td>
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#### Instructional Targets

<table>
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<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
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</tbody>
</table>

#### Assessment Types

- Students should be able to:
  - Convert between numerical forms as appropriate.
  - Apply properties of operations to calculate with numbers in any form.
  - Assess the reasonableness of answers using mental computation and estimation strategies.
  - Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form.

#### Explanations and Examples

Estimation strategies for calculations with fractions and decimals extend from students’ work with whole number operations. Estimation strategies include, but are not limited to:

- Front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts).
- Clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate).
- Rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values).
- Using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000).
- Using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

**Example:**

- The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

  \[
  \begin{array}{c|c|c}
  \times & \times & 11 \\
  \hline
  52 & 41 & 520.5 \\
  \end{array}
  \]
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.EE.4</th>
<th>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DESCRIPTION</strong></td>
<td>Students write an equation or inequality to model the situation. Students explain how they determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution. In contextual problems, students define the variable and use appropriate units. Students solve multi-step equations derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution.</td>
</tr>
</tbody>
</table>

**Example 1:**
The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

**Solution:**
\[
x = \text{cost of one pass}
\]

![Equation](image)

**Example 2:**
Amy had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost including tax?

**Solution:**
\[
x = \text{number of pens}
\]
\[
26 = 14.30 + 10x
\]
Solving for \(x\) gives $1.17 for each pen.
Example 4:
The sum of three consecutive even numbers is 48. What is the smallest of these numbers?

Solution:
\[ x = \text{the smallest even number} \]
\[ x + 2 = \text{the second even number} \]
\[ x + 4 = \text{the third even number} \]

\[ x + x + 2 + x + 4 = 48 \]
\[ 3x + 6 = 48 \]
\[ 3x = 42 \]
\[ x = 14 \]

Example 5:
Solve: \[ x + 3 = -5 \]

Solution:
\[ x = 7 \]

Students solve and graph inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context.

Example 1:
Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.

Solution:
\[ x = \text{cost of one t-shirt} \]
\[ 8x + 22 \leq 60 \]
\[ x = 4.75 \]
\[ 4 \text{ is the most t-shirts she can purchase} \]

Example 2:
Steven has $25 dollars to spend. He spent $10.81, including tax, to buy a new DVD. He needs to save $10.00 but he wants to buy a snack. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy?

Solution:
\[ x = \text{number of packages of peanuts} \]
\[ 25 \geq 10.81 + 10.00 + 0.38x \]
\[ x = 11.03 \]
\[ \text{Steven can buy 11 packages of peanuts} \]
### DESCRIPTION (continued)

Example 3:
7 - x > 5.4

**Solution:**

x < 1.6

Example 4:
Solve -0.5x - 5 < -1.5 and graph the solution on a number line.

**Solution:**

x > -7

---

### ESSENTIAL QUESTION(S)

What algebraic strategies can be used to solve problems when reasoning about the quantities?

### MATHEMATICAL PRACTICE(S)

- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment

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</table>
### SUBSTANDARD DECONSTRUCTED

#### 7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

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<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Identify the sequence of operations used to solve an algebraic equation of the form $px + q = r$ and $p(x + q) = r$. Fluently solve equations of the form $px + q = r$ and $p(x + q) = r$ with speed and accuracy.</td>
<td>Compare an algebraic solution to an arithmetic solution by identifying the sequence of the operations used in each approach. Use variables and construct equations to represent quantities of the form $px + q = r$ and $p(x + q) = r$ from real-world and mathematical problems. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers.</td>
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</tbody>
</table>

#### 7.EE.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p$, $q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

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<tr>
<td>Students should be able to:</td>
<td>Graph the solution set of the inequality of the form $px + q &gt; r$ or $px + q &lt; r$, where $p$, $q$, and $r$ are specific rational numbers.</td>
<td>Interpret the solution set of an inequality in the context of the problem. Solve word problems leading to inequalities of the form $px + q &gt; r$ or $px + q &lt; r$, where $p$, $q$, and $r$ are specific rational numbers.</td>
<td></td>
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</table>
Examples:

- Amie had $26 to spend on school supplies. After buying 10 pens, she had $14.30 left. How much did each pen cost?
- The sum of three consecutive even numbers is 48. What is the smallest of these numbers?

Solve: \( \frac{5}{4}n + 5 = D \)

- Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.
- Steven has $25. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy?

Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution.

Solve: \( \frac{1}{2}x + 3 > 2 \) and graph your solution on a number line.
DOMAIN:

GEOMETRY (G)

SEVENTH GRADE
MATHEMATICS
## Geometry (G)

### Clusters

1. Draw, construct and describe geometrical figures and describe the relationships between them.
2. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

### Geometry

#### Triangles and Transformations

<table>
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<th>Sixth</th>
<th>Seventh</th>
<th>Eighth</th>
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<tbody>
<tr>
<td>Similarity and Congruence, including Constructions and Transformations</td>
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<td>Similarity and Congruence, including Constructions and Transformations</td>
</tr>
</tbody>
</table>

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

- **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- **8.G.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

- **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

- **8.G.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

- **8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

- **8.G.4** Verify experimentally the properties of rotations, reflections, and translations.

- **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

### Length, Area, and Volume

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<tr>
<th>Sixth</th>
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<th>Eighth</th>
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<tbody>
<tr>
<td>Area and Volume of Geometrical Shapes and Solids</td>
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</tbody>
</table>

- **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas \( V = l w h \) and \( V = b h \) to find volumes of right.

- **7.G.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

- **6.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

- **7.G.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

- **8.G.1** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
## GEOMETRY (G)

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<td><strong>LENGTH, AREA, AND VOLUME</strong></td>
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<tr>
<td>Area and Volume of Geometrical Shapes and Solids</td>
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<tr>
<td>7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</td>
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<tr>
<td>6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</td>
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<tr>
<td><strong>SHAPES AND ANGLES</strong></td>
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<tr>
<td>Angles</td>
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<tr>
<td>7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</td>
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</table>
## SEVENTH GRADE

**LEXILE GRADE LEVEL BANDS: 970L TO 1120L**

<table>
<thead>
<tr>
<th>CLUSTER</th>
<th>1. Draw, construct and describe geometrical figures and describe the relationships between them.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIG IDEA</td>
<td>- Proportionality is a numerical relationship that forms a straight line on the coordinate graph.</td>
</tr>
<tr>
<td>ACADEMIC VOCABULARY</td>
<td>scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, perpendicular</td>
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</table>

### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.G.1</th>
<th><strong>Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</strong></th>
</tr>
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</table>

**DESCRIPTION**

Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

**Example:**

If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?

![Rectangle](image)

**Solution:**

The perimeter is linear or one-dimensional. Multiply the perimeter of the given rectangle (18 in.) by the scale factor (1.5) to give an answer of 27 in. Students could also increase the length and width by the scale factor of 1.5 to get 10.5 in. for the length and 3 in. for the width. The perimeter could be found by adding $10.5 + 10.5 + 3 + 3$ to get 27 in.

The area is two-dimensional so the scale factor must be squared. The area of the new rectangle would be $14 \times 1.5^2$ or 31.5 in².
### MATHEMATICS

#### ESSENTIAL QUESTION(S)
How is the technique of scale drawing an efficient way to solve problems?

#### MATHEMATICAL PRACTICE(S)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

#### DOK Range Target for Instruction & Assessment
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#### Instructional Targets
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<td></td>
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</table>

#### Students should be able to:
- Use ratios and proportions to create scale drawing.
- Identify corresponding sides of scaled geometric figures.
- Compute lengths and areas from scale drawings using strategies such as proportions.
- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- Reproduce a scale drawing that is proportional to a given geometric figure using a different scale.

#### EXPLANATIONS AND EXAMPLES
**Example:**
- Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie’s room? Reproduce the drawing at 3 times its current size.

```
4 cm  5.6 cm
1.2 cm
1.2 cm
4.4 cm
2.8 cm
```
### STANDARD AND DECONSTRUCTION

| 7.G.2 | Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. |

**DESCRIPTION**

Students draw geometric shapes with given parameters. Parameters could include parallel lines, angles, perpendicular lines, line segments, etc.

**Example 1:**

Draw a quadrilateral with one set of parallel sides and no right angles.

Students understand the characteristics of angles and side lengths that create a unique triangle, more than one triangle or no triangle.

**Example 2:**

Can a triangle have more than one obtuse angle? Explain your reasoning.

**Example 3:**

Will three sides of any length create a triangle? Explain how you know which will work.

Possibilities to examine are:

- a. 13 cm, 5 cm, and 6 cm
- b. 3 cm, 3 cm, and 3 cm
- c. 2 cm, 7 cm, 6 cm

**Solution:**

"A" above will not work; "B" and "C" will work. Students recognize that the sum of the two smaller sides must be larger than the third side.
Why is it important to draw a figure using accurate conditions when examining geometric shapes?

7.MP.5. Use appropriate tools strategically.
7.MP.6. Attend to precision.
7.MP.7. Look for and make use of structure.
7.MP.8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTION(S)

Why is it important to draw a figure using accurate conditions when examining geometric shapes?

MATHEMATICAL PRACTICE(S)

7.MP.5. Use appropriate tools strategically.
7.MP.6. Attend to precision.
7.MP.7. Look for and make use of structure.
7.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment

Explanations and Examples

Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity.

Examples:

Is it possible to draw a triangle with a 90° angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?

- Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not?
- Draw an isosceles triangle with only one 80 degree angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?

- Can you draw a triangle with sides that are 13 cm, 5 cm and 6 cm?
- Draw a quadrilateral with one set of parallel sides and no right angles.
# SEVENTH GRADE

## LEXILE GRADE LEVEL BANDS: 970L TO 1120L

### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.G.3</th>
<th>Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</th>
</tr>
</thead>
</table>

#### DESCRIPTION

Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram.

![Diagram](http://intermath.coe.uga.edu/dictnary/descript.asp?termID=95)

The major emphasis for measurement in Grade 5 is volume. Volume not only introduces a third dimension and thus a significant challenge to students’ spatial structuring, but also complexity in the nature of the materials measured. That is, solid units are “packed,” such as cubes in a three-dimensional array, whereas a liquid “fills” three-dimensional space, taking the shape of the container. The unit structure for liquid measurement may be psychologically one dimensional for some students.

- If the pyramid is cut with a plane (green) parallel to the base, the intersection of the pyramid and the plane is a square cross section (red).

- If the pyramid is cut with a plane (green) passing through the top vertex and perpendicular to the base, the intersection of the pyramid and the plane is a triangular cross section (red).

- If the pyramid is cut with a plane (green) perpendicular to the base, but not through the top vertex, the intersection of the pyramid and the plane is a trapezoidal cross section (red).

http://intermath.coe.uga.edu/dictnary/descript.asp?termID=95
## Mathematics

### Essential Question(s)
What is the importance of examining the 2-D figures inside of any 3-D shape?

### Mathematical Practice(s)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment

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<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
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**Students should be able to:**
- Define “slicing” as the cross-section of a 3-D figure.
- Describe the two-dimensional figures that result from slicing a three-dimensional figure such as a right rectangular prism or pyramid.
- Analyze three-dimensional shapes by examining two-dimensional cross-sections.

### Explanations and Examples

**Example:**
- Using a clay model of a rectangular prism, describe the shapes that are created when planar cuts are made diagonally, perpendicularly, and parallel to the base.

![Clay model of a rectangular prism with cuts](image)
2. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Big Idea

- Area is an attribute of a 2-D figure (that can describe patterns and reason in the physical world) that can be measured.

Academic Vocabulary

inscribed, circumference, radius, diameter, \( \pi \), supplementary, vertical, adjacent, complementary, pyramids, face, base

Standard and Deconstruction

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between circumference and area of a circle.

Description

Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as \( \pi \). Building on these understandings, students generate the formulas for circumference and area.

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is the circumference \((2\pi r)\). The area of the rectangle (and therefore the circle) is found by the following calculations:

\[
\text{Area} = \frac{1}{4} (2\pi r) \times r \\
\text{Area} = \pi r \times r \\
\text{Area} = \pi r^2
\]

http://mathworld.wolfram.com/Circle.html

Students solve problems (mathematical and real-world) involving circles or semi-circles.

Note: Because \( \pi \) is an irrational number that neither repeats nor terminates, the measurements are approximate when 3.14 is used in place of \( \pi \).

Example 1:
The center of the circle is at (5, -5). What is the area of the circle?

\[
\begin{array}{c}
\frac{1}{2} \\
\end{array}
\]

Solution:
The radius of the circle is 4. Using the formula, \( \text{Area} = \pi r^2 \), the area of the circle is approximately 50.24 units\(^2\).

Students build on their understanding of area from 6th grade to find the area of left-over materials when circles are cut from squares and triangles or when squares and triangles are cut from circles.
Example 2:
If a circle is cut from a square piece of plywood, how much plywood would be left over?

Solution:
The area of the square is 28 x 28 or 784 in². The diameter of the circle is equal to the length of the side of the square, or 28", so the radius would be 14". The area of the circle would be approximately 615.44 in². The difference in the amounts (plywood left over) would be 168.56 in² (784 – 615.44).

Example 3:
What is the perimeter of the inside of the track?

Solution:
The ends of the track are two semicircles, which would form one circle with a diameter of 62m. The circumference of this part would be 194.68 m. Add this to the two lengths of the rectangle and the perimeter is 2194.68 m.

“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of why the formula works and how the formula relates to the measure (area and circumference) and the figure. This understanding should be for all students.
### Essential Question(s)
What are efficient strategies for solving area and circumference of a circle?

### Mathematical Practice(s)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment

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<tr>
<td>Know the parts of a circle including radius, diameter, area, circumference, center, and chord.</td>
<td>Justify that pi (π) can be derived from the circumference and diameter of a circle.</td>
<td>Apply circumference or area formulas to solve mathematical and real-world problems.</td>
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<tr>
<td>Identify pi (π).</td>
<td>Justify the formulas for area and circumference of a circle and how they relate to pi (π).</td>
<td>Informally derive the relationship between circumference and area of a circle.</td>
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<tr>
<td>Know the formulas for area and circumference of a circle.</td>
<td>Given the circumference of a circle, find its area.</td>
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<tr>
<td>Given the area of a circle, find its circumference.</td>
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</table>

### Explanations and Examples
- **Examples:**
  - The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?
  - Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures. Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles.
  - Students will use a circle as a model to make several equal parts as you would in a pie model. The greater number the cuts, the better. The pie pieces are laid out to form a shape similar to a parallelogram. Students will then write an expression for the area of the parallelogram related to the radius (note: the length of the base of the parallelogram is half the circumference, or \( \pi r \), and the height is \( r \), resulting in an area of \( \pi r^2 \). Extension: If students are given the circumference of a circle, could they write a formula to determine the circle's area or given the area of a circle, could they write the formula for the circumference?
# 7.G.5

## Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

### Description

Students use understandings of angles and deductive reasoning to write and solve equations.

**Example:**

Find the measure of angle $b$.

![Diagram](image)

Note: Not drawn to scale.

**Solution:**

Because the 45°, 50° angles and $b$ form a supplementary angle, the measure of angle $b$ would be 85°. The measures of the angles of a triangle equal 180° so $75° + 85° + a = 180°$. The measure of angle $a$ would be 20°.

### Essential Question(s)

What are efficient strategies for finding an unknown angle in a figure?

### Mathematical Practice(s)

- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment

- **1**
- **2**
- **3**
- **4**

### Instructional Targets

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<td><strong>Students should be able to:</strong></td>
<td>Identify and recognize types of angles: supplementary, complementary, vertical, adjacent. Determine complements and supplements of a given angle.</td>
<td>Determine unknown angle measures by writing and solving algebraic equations based on relationships between angles.</td>
<td>Tasks assessing modeling/applications.</td>
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</tbody>
</table>
Angle relationships that can be explored include but are not limited to:
- Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Examples:
- Write and solve an equation to find the measure of angle $x$.

![Diagram with angles $x$, $50^\circ$, and $120^\circ$.]
- Write and solve an equation to find the measure of angle $x$. 

![Diagram with angles $x$, $120^\circ$, and $50^\circ$.]
### STANDARD AND DECONSTRUCTION

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<td>7.G.6</td>
<td>Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
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</table>

**DESCRIPTION**

Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects. (Composite shapes) Students will not work with cylinders, as circles are not polygons. At this level, students determine the dimensions of the figures given the area or volume.

“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of why the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for all students.

Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume.

Students solve for missing dimensions, given the area or volume.

**Example 1:**

A triangle has an area of 6 square feet. The height is four feet. What is the length of the base?

**Solution:**

One possible solution is to use the formula for the area of a triangle and substitute in the known values, then solve for the missing dimension. The length of the base would be 3 feet.

**Example 2:**

The surface area of a cube is 96 in². What is the volume of the cube?

**Solution:**

The area of each face of the cube is equal. Dividing 96 by 6 gives an area of 16 in² for each face. Because each face is a square, the length of the edge would be 4 in. The volume could then be found by multiplying 4 x 4 x 4 or 64 in³.
Example 3:
Huong covered the box to the right with sticky-backed decorating paper. The paper costs 3¢ per square inch. How much money will Huong need to spend on paper?

![Box with dimensions](image)

**Solution:**
The surface area can be found by using the dimensions of each face to find the area and multiplying by 2:

- Front: 7 in. x 9 in. = 63 in² x 2 = 126 in²
- Top: 3 in. x 7 in. = 21 in² x 2 = 42 in²
- Side: 3 in. x 9 in. = 27 in² x 2 = 54 in²

The surface area is the sum of these areas, or 222 in². If each square inch of paper costs $0.03, the cost would be $6.66.

Example 4:
Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism (see diagram below). If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box? How much packaging material was used to construct the cracker box? Explain how you got your answer.

**Solution:**
Volume can be calculated by multiplying the area of the base (triangle) by the height of the prism. Substitute given values and solve for the area of the triangle.

\[ V = Bh \]

To find the height of the triangle, use the area formula for the triangle, substituting the known values in the formula and solving for height. The height of the triangle is 12 cm.

The problem also asks for the surface area of the package. Find the area of each face and add:

- 2 triangular bases: \( \frac{1}{2} \times 18 \text{ cm} \times 12 \text{ cm} = 108 \text{ cm}^2 \times 2 = 216 \text{ cm}^2 \)
- 2 rectangular faces: \( 15 \text{ cm} \times 30 \text{ cm} = 450 \text{ cm}^2 \times 2 = 900 \text{ cm}^2 \)
- 1 rectangular face: \( 18 \text{ cm} \times 30 \text{ cm} = 540 \text{ cm}^2 \)

Adding 216 cm² + 900 cm² + 540 cm² gives a total surface area of 1656 cm².
### Mathematics

#### Essential Question(s)
What are efficient strategies for solving problems involving area, volume and surface areas of 2-D and 3-D objects?

#### Mathematical Practice(s)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

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<td>Students should be able to:</td>
<td>Know the formulas for area and volume and the procedure for finding surface area and when to use them in real-world and math problems.</td>
<td>Solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
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</table>
Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations.

**Examples:**

- Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?

- A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.) Make a poster explaining your work to share with the class.

- Find the area of a triangle with a base length of three units and a height of four units.

- Find the area of the trapezoid shown below using the formulas for rectangles and triangles.
STATISTICS AND PROBABILITY (SP)
** DOMAIN **  
Statistics and Probability (SP)

** CLUSTERS **
1. Use random sampling to draw inferences about a population.  
2. Draw informal comparative inferences about two populations.  
3. Investigate chance processes and develop, use, and evaluate probability models.

### STATISTICS AND PROBABILITY (SP)

#### VARIATION, DISTRIBUTION, AND MODELING

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<td>7.SP.1</td>
<td>Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</td>
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<td>7.SP.2</td>
<td>Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.</td>
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<td><strong>Describing the Distribution of Two Data Sets</strong></td>
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<td>7.SP.3</td>
<td>Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</td>
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<td>7.SP.4</td>
<td>Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</td>
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<td>8.SP.3</td>
<td>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.</td>
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### CHANCE AND PROBABILITY

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<tr>
<td>7.SP.5</td>
<td>Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
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<td>6.G.2</td>
<td>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = 1wh and V = bh to find volumes of right.</td>
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<tr>
<td>7.G.3</td>
<td>Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</td>
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## STATISTICS AND PROBABILITY

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### CLUSTER
1. Use random sampling to draw inferences about a population.

### BIG IDEA
- General conclusions can be made about a set of data from an appropriate set of questions.

### ACADEMIC VOCABULARY
random sampling, population, representative sample, inferences

### STANDARD AND DECONSTRUCTION

#### 7.SP.1
Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

#### DESCRIPTION
Students recognize that it is difficult to gather statistics on an entire population. Instead a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.

#### ESSENTIAL QUESTION(S)
Why is it important to select an appropriate sample to make an inference?

#### MATHEMATICAL PRACTICE(S)
7.MP.3. Construct viable arguments and critique the reasoning of others.
7.MP.6. Attend to precision.

#### DOK Range Target for Instruction & Assessment
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#### Instructional Targets

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#### Students should be able to:
- Know statistics terms such as population, sample, sample size, random sampling, generalizations, valid, biased and unbiased.
- Recognize sampling techniques such as convenience, random, systematic, and voluntary.
- Know that generalizations about a population from a sample are valid only if the sample is representative of that population.
- Apply statistics to gain information about a population from a sample of the population.
- Generalize that random sampling tends to produce representative samples and support valid inferences.
### Example:

- The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students’ preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why?

1. Write all of the students’ names on cards and pull them out in a draw to determine who will complete the survey.
2. Survey the first 20 students that enter the lunch room.
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.SP.2</th>
<th>Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DESCRIPTION</strong></td>
<td>Students collect and use multiple samples of data to make generalizations about a population. Issues of variation in the samples should be addressed.</td>
</tr>
<tr>
<td><strong>ESSENTIAL QUESTION(S)</strong></td>
<td>Why is it important to select an appropriate sample to make an inference?</td>
</tr>
</tbody>
</table>
| **MATHEMATICAL PRACTICE(S)** | 7.MP.1. Make sense of problems and persevere in solving them.  
7.MP.2. Reason abstractly and quantitatively.  
7.MP.3. Construct viable arguments and critique the reasoning of others.  
7.MP.5. Use appropriate tools strategically.  
7.MP.6. Attend to precision.  
7.MP.7. Look for and make use of structure. |
| **DOK Range Target for Instruction & Assessment** | ☐ 1 ✗ 2 ✗ 3 ☐ 4 |
| **Instructional Targets** | **Know: Concepts/Skills** | **Think** | **Do** |
| Assessment Types | Tasks assessing concepts, skills, and procedures. | Tasks assessing expressing mathematical reasoning. | Tasks assessing modeling/applications. |
| Students should be able to: | Define random sample. | Analyze and interpret data from a random sample to draw inferences about a population with an unknown characteristic of interest. |  
Identify an appropriate sample size. | Generate multiple samples (or simulated samples) of the same size to determine the variation in estimates or predictions by comparing and contrasting the samples. |
**Example:**
- Below is the data collected from two random samples of 100 students regarding students’ school lunch preferences. Make at least two inferences based on the results.

| Lunch Preferences |
|---|---|---|---|
| student sample | hamburgers | tacos | pizza | total |
| #1 | 12 | 14 | 74 | 100 |
| #2 | 12 | 11 | 77 | 100 |

**Solution:**
- Most students prefer pizza.
- More people prefer pizza than hamburgers and tacos combined.
### Cluster 2. Draw informal comparative inferences about two populations.

#### Big Idea
- Appropriate measures of center and spread can generate general conclusions about a set of data.

#### Academic Vocabulary
- variation/variability, distribution, measures of center, measures of variability

#### Standard and Deconstruction

| 7.SP.3 | Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. |

#### Description
This is the students' first experience with comparing two data sets. Students build on their understanding of graphs, mean, median, Mean Absolute Deviation (MAD) and interquartile range from 6th grade. Students understand that:

1. A full understanding of the data requires consideration of the measures of variability, as well as, mean or median,
2. Variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap, and
3. Median is paired with the interquartile range and mean is paired with the mean absolute deviation.
## Statistics & Probability

**Essential Question(s)**
What statistical processes help gain understanding about the relationship of two populations?

**Mathematical Practice(s)**
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.

### DOK Range Target for Instruction & Assessment

<table>
<thead>
<tr>
<th>DOK</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

### Instructional Targets

#### Assessment Types

- **Know: Concepts/Skills**
  - Tasks assessing concepts, skills, and procedures.

- **Think**
  - Tasks assessing expressing mathematical reasoning.

- **Do**
  - Tasks assessing modeling/applications.

#### Students should be able to:
- Identify measures of central tendency (mean, median, and mode) in a data distribution.
- Identify measures of variation including upper quartile, lower quartile, upper extreme-maximum, lower extreme-minimum, range, interquartile range, and mean absolute deviation.
- Compare two numerical data distributions on a graph by visually comparing data displays, and assessing the degree of visual overlap.
- Compare the differences in the measure of central tendency in two numerical data distributions by measuring the difference between the centers and expressing it as a multiple of a measure of variability.
Students can readily find data as described in the example on sports teams or college websites. Other sources for data include American Fact Finder (Census Bureau), Fed Stats, Ecology Explorers, USGS, or CIA World Factbook. Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

Example:

Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn’t know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists.

Basketball Team – Height of Players in inches for 2010-2011 Season
75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84

Soccer Team – Height of Players in inches for 2010
73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

To compare the data sets, Jason creates two dot plots on the same scale. The shortest player is 65 inches and the tallest players are 84 inches.

In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation to compare the data sets. Jason sets up a table for each data set to help him with the calculations.

The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches.

The mean absolute deviation (MAD) is calculated by taking the mean of the absolute deviations for each data point. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set.

The mean absolute deviation is 2.53 inches for the basketball players and 2.14 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams is approximately 3 times the variability of the data sets (7.68 ÷ 2.53 = 3.04).
### Soccer Players (n = 29)

<table>
<thead>
<tr>
<th>Height (in)</th>
<th>Deviation from Mean (in)</th>
<th>Absolute Deviation (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>-7</td>
<td>7</td>
</tr>
<tr>
<td>67</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>69</td>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>69</td>
<td>-3</td>
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</tr>
<tr>
<td>69</td>
<td>-3</td>
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<td>71</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>72</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

### Basketball Players (n = 16)

<table>
<thead>
<tr>
<th>Height (in)</th>
<th>Deviation from Mean (in)</th>
<th>Absolute Deviation (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>-7</td>
<td>7</td>
</tr>
<tr>
<td>75</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>76</td>
<td>-4</td>
<td>4</td>
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<tr>
<td>78</td>
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</tr>
<tr>
<td>81</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>82</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
### SEVENTH GRADE

**LEXILE GRADE LEVEL BANDS: 970L TO 1120L**

#### EXPLANATIONS AND EXAMPLES (continued)

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>73</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>73</td>
<td>+1</td>
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</tr>
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<tr>
<td>76</td>
<td>+4</td>
<td>4</td>
</tr>
<tr>
<td>78</td>
<td>+6</td>
<td>6</td>
</tr>
<tr>
<td>Σ = 2090</td>
<td></td>
<td>Σ = 62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>4</td>
<td>4</td>
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<td>4</td>
</tr>
<tr>
<td>84</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Σ = 1276</td>
<td></td>
<td>Σ = 40</td>
</tr>
</tbody>
</table>

Mean = \( \frac{2090}{29} \approx 72 \text{ inches} \)

MAD = \( \frac{62}{29} \approx 2.14 \) inches

Mean = \( \frac{1276}{16} \approx 80 \text{ inches} \)

MAD = \( \frac{40}{16} \approx 2.53 \) inches
7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

DESCRIPTION

Students compare two sets of data using measures of center (mean and median) and variability (MAD and IQR). Showing the two graphs vertically, rather than side by side, helps students make comparisons. For example, students would be able to see from the display of the two graphs that the 'ideal scores' are generally higher than the 'organization scores'. One observation students might make is that the scores for organization are clustered around a score of 3, whereas the scores for ideas are clustered around a score of 5.
**ESSENTIAL QUESTION(S)**

What statistical processes help gain understanding about the relationship of two populations?

**MATHEMATICAL PRACTICE(S)**

- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

- 1
- 2
- 3
- 4

**Instructional Targets**

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
<td></td>
</tr>
</tbody>
</table>

**Students should be able to:**

- Find measures of central tendency (mean, median, and mode) and measures of variability (range, quartile, etc.).
- Analyze and interpret data using measures of central tendency and variability.
- Draw informal comparative inferences about two populations from random samples.

**EXPLANATIONS AND EXAMPLES**

Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.

**Example:**

- The two data sets below depict random samples of the housing prices sold in the King River and Toby Ranch areas of Arizona. Based on the prices below, which measure of center will provide the most accurate estimation of housing prices in Arizona? Explain your reasoning.
- King River area: {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}
- Toby Ranch homes: {5 million, 154000, 250000, 250000, 200000, 160000, 190000}
### Cluster
Investigate chance processes and develop, use, and evaluate probability models.

### Big Idea
- The chance of an event occurring can be described numerically and can be used to make predictions.

### Academic Vocabulary
bar graph, certainty, circle graph, cross section, collection, coordinates, direction, distribution, increments, Venn diagram, matrix, matrices, median, mode, random, standard deviation, survey, probability, arrangement, likelihood, intervals, statistical

### Standard and Deconstruction

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.SP.5</td>
<td>Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</td>
</tr>
</tbody>
</table>

**Description**
This is the students’ first formal introduction to probability.

Students recognize that the probability of any single event can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1, inclusive, as illustrated on the number line below.

**Example:**
There are three choices of jellybeans – grape, cherry, and orange. If the probability of getting a grape is 3/10 and the probability of getting cherry is 1/5, what is the probability of getting orange?

**Solution:**
The combined probabilities must equal 1. The combined probability of grape and cherry is 5/10. The probability of orange must equal 5/10 to get a total of 1.
**ESSENTIAL QUESTION(S)**

What is the importance of 0 and 1 when examining the probability of an event?

**MATHEMATICAL PRACTICE(S)**

7.MP.5. Use appropriate tools strategically.
7.MP.6. Attend to precision.
7.MP.7. Look for and make use of structure.

**DOK Range Target for Instruction & Assessment**

[ ] 1  [ ] 2  [ ] 3  [ ] 4

**Instructional Targets**

<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
</table>
| Tasks assessing concepts, skills, and procedures. | Tasks assessing expressing mathematical reasoning. | Tasks assessing modeling/applications. | Draw conclusions to determine that a greater likelihood occurs as the number of favorable outcomes approaches the total number of outcomes.

**Students should be able to:**

- Know that probability is expressed as a number between 0 and 1.
- Know that a random event with a probability of 1/2 is equally likely to happen.
- Know that as probability moves closer to 1, it is increasingly likely to happen.
- Know that as probability moves closer to 0, it is decreasingly likely to happen.

**Explanations and Examples**

Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line. Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

Example:

The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.
Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line. Students can use simulations such as Marble Mania on AAAS or the Random Drawing Tool on NCTM’s Illuminations to generate data and examine patterns.

Marble Mania [http://www.sciencenetlinks.com/interactives/marble/marblemania.html]

Example:

- The container below contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.
### Standard and Deconstruction

| 7.SP.6 | Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. |

#### Description

Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability. The focus of this standard is relative frequency. The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful events, expressed as the value calculated by dividing the number of times an event occurs by the total number of times an experiment is carried out.

**Example 1:**
Suppose we toss a coin 50 times and have 27 heads and 23 tails. We define a head as a success. The relative frequency of heads is:

\[
\frac{27}{50} = 54\%
\]

The probability of a head is 50%. The difference between the relative frequency of 54% and the probability of 50% is due to small sample size.

The probability of an event can be thought of as its long-run relative frequency when the experiment is carried out many times.

**Example 2:**
A bag contains 100 marbles, some red and some purple. Suppose a student, without looking, chooses a marble out of the bag, records the color, and then places that marble back in the bag. The student has recorded 9 red marbles and 11 purple marbles. Using these results, predict the number of red marbles in the bag.

(Adapted from SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do.*)
### ESSENTIAL QUESTION(S)
What efficient strategies can be used to help determine the probability of a chance event?

### MATHEMATICAL PRACTICE(S)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.

### DOK Range Target for Instruction & Assessment
- ☐ 1
- 🔧 2
- ☑️ 3
- ☐ 4

### Instructional Targets
<table>
<thead>
<tr>
<th>Assessment Types</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Determine relative frequency (experimental probability) which is the number of times an outcome occurs divided by the total number of times the experiment is completed.</td>
<td>Determine the relationship between experimental and theoretical probabilities by using the law of large numbers. Predict the relative frequency (experimental probability) of an event based on its theoretical probability.</td>
<td>Tasks assessing expressing mathematical reasoning. Tasks assessing modeling/applications.</td>
</tr>
</tbody>
</table>

### Students should be able to:
- Students can collect data using physical objects or graphing calculator or web-based simulations.
- Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.

### EXPLANATIONS AND EXAMPLES
Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.

**Example:**
Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities (How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?).

Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. (An example would be 3 green marbles, 6 blue marbles, 3 blue marbles.)

Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.
### STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.SP.7</th>
<th>Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</th>
</tr>
</thead>
</table>

#### DESCRIPTION

Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

**Example 1:**
Jason is tossing a fair coin. He tosses the coin ten times and it lands on heads eight times. If Jason tosses the coin an eleventh time, what is the probability that it will land on heads?

**Solution:**
The probability would be $1/2$. The result of the eleventh toss does not depend on the previous results.

**Example 2:**
Devise an experiment using a coin to determine whether a baby is a boy or a girl. Conduct the experiment ten times to determine the gender of ten births. How could a number cube be used to simulate whether a baby is a girl or a boy or girl?

**Example 3:**
Conduct an experiment using a Styrofoam cup by tossing the cup and recording how it lands.
- How many trials were conducted?
- How many times did it land right side up?
- How many times did it land upside down?
- How many times did it land on its side?
- Determine the probability for each of the above results.
### Essential Question(s)
What efficient strategies can be used to help determine the probability of events?

### Mathematical Practice(s)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.3. Construct viable arguments and critique the reasoning of others.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment
- 1
- 2
- 3
- 4

### Substandard Deconstructed

#### 7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

<table>
<thead>
<tr>
<th>Instructional Targets</th>
<th>Know: Concepts/Skills</th>
<th>Think</th>
<th>Do</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
</tr>
<tr>
<td>Students should be able to:</td>
<td>Recognize uniform (equally likely) probability. Use models to determine the probability of events.</td>
<td>Analyze a probability model and justify why it is uniform. Develop a uniform probability model and use it to determine the probability of each outcome/event.</td>
<td></td>
</tr>
</tbody>
</table>

#### 7.SP.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

<table>
<thead>
<tr>
<th>Instructional Targets</th>
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<th>Think</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Assessment Types</td>
<td>Tasks assessing concepts, skills, and procedures.</td>
<td>Tasks assessing expressing mathematical reasoning.</td>
<td>Tasks assessing modeling/applications.</td>
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<tr>
<td>Students should be able to:</td>
<td>Use models to determine the probability of events.</td>
<td>Analyze a probability model and explain any discrepancies. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.</td>
<td></td>
</tr>
</tbody>
</table>
**Explanations and Examples**

Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data. Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web-based simulations. Students can also develop models for geometric probability (i.e. a target).

Example:
- If you choose a point in the square, what is the probability that it is not in the circle?

![Diagram](image-url)
## STANDARD AND DECONSTRUCTION

<table>
<thead>
<tr>
<th>7.SP.8</th>
<th>Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DESCRIPTION</strong></td>
<td>Students use tree diagrams, frequency tables, and organized lists, and simulations to determine the probability of compound events.</td>
</tr>
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</table>

**Example 1:**
How many ways could the 3 students, Amy, Brenda, and Carla, come in 1st, 2nd and 3rd place?

**Solution:**
Making an organized list will identify that there are 6 ways for the students to win a race.

A, B, C  
A, C, B  
B, C, A  
B, A, C  
C, A, B  
C, B, A

**Example 2:**
A fair coin will be tossed three times. What is the probability that two heads (H) and one tail (T), in any order, will result?

(Adapted from SREB publication Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do.)

**Solution:**
HHT, HTH and THH, so the probability would be 3/8.
### Essential Question(s)
What efficient strategies can be used to help determine the likelihood of compound events to occur?

### Mathematical Practice(s)
- 7.MP.1. Make sense of problems and persevere in solving them.
- 7.MP.2. Reason abstractly and quantitatively.
- 7.MP.5. Use appropriate tools strategically.
- 7.MP.7. Look for and make use of structure.
- 7.MP.8. Look for and express regularity in repeated reasoning.

### DOK Range Target for Instruction & Assessment
- 1
- 2
- 3
- 4

### Substandard Deconstructed

#### 7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

<table>
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<tr>
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<th>Do</th>
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<td>Students should be able to:</td>
<td>Define and describe a compound event. Know that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</td>
<td>Find probabilities of compound events using organized lists, tables, tree diagrams, etc. and analyze the outcomes.</td>
<td></td>
</tr>
</tbody>
</table>

#### 7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g. “rolling double sixes”), identify the outcomes in the sample space which compose the event.

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<tr>
<td>Students should be able to:</td>
<td>Identify the outcomes in the sample space for an everyday event.</td>
<td>Choose the appropriate method such as organized lists, tables and tree diagrams to represent sample spaces for compound events.</td>
<td></td>
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Examples:

- Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another. What is the sample space for this situation? Explain how you determined the sample space and how you will use it to find the probability of drawing one blue marble followed by another blue marble.

- Show all possible arrangements of the letters in the word FRED using a tree diagram. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your “word” will have an F as the first letter?